

Closing Wed: HW\_3A,3B,3C (6.1-6.3)

Exam 1 is Thursday, April 20<sup>th</sup> in your normal quiz section.

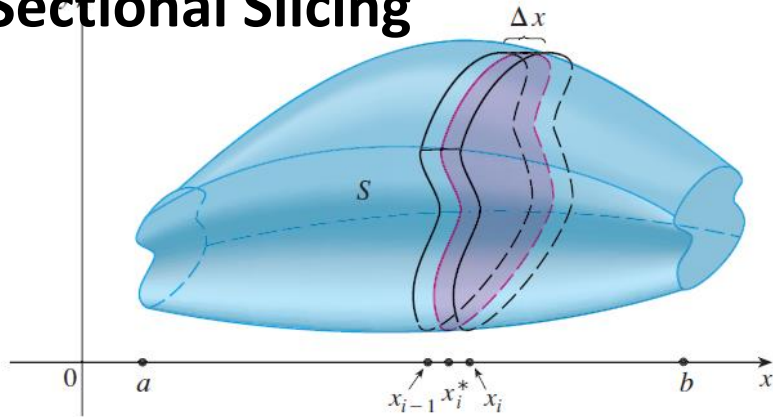
It covers 4.9, 5.1-5.5, 6.1-6.3

*Entry Task:*

Find the area of the region bounded by  $4x = y^2$  and  $y = 2x^3$  in 2 ways:

- (i) Using  $dx$
- (ii) Using  $dy$

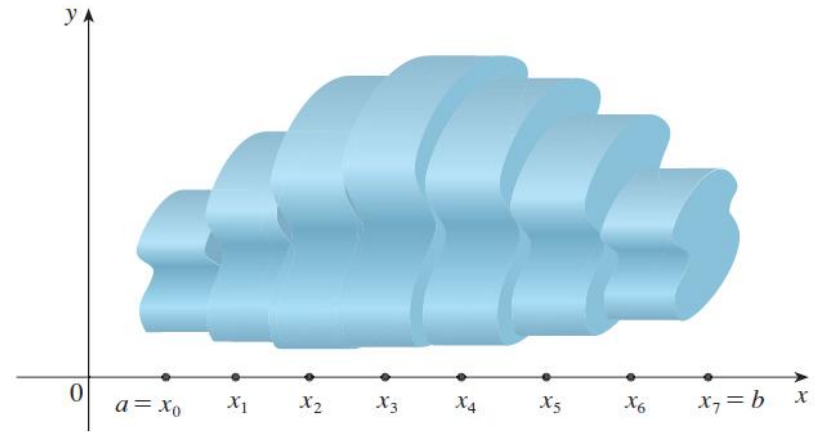
## 6.2 Finding Volumes Using Cross-Sectional Slicing



If we can find the general formula,  $A(x_i)$ , for the area of a cross-sectional slice, then we can approximate volume by:

Volume of one slice  $\approx A(x_i) \Delta x$

Total Volume  $\approx \sum_{i=1}^n A(x_i) \Delta x$



This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n \rightarrow \infty} \sum_{i=1}^n A(x_i) \Delta x$$

We conclude

$$\text{Volume} = \int_a^b A(x) dx =$$

$$\int_a^b \text{"Cross-sectional area formula"} dx$$

## Volume using cross-sectional slicing

1. Draw and label! Cut **perpendicular** to the axis of rotation. Draw a *typical* cross-section.

Label  $x$  or  $y$  and  $\Delta x$  or  $\Delta y$ .

Label **everything** in terms of the appropriate variable.

2. Formula for cross-sectional area?

*disc:* Area =  $\pi(\text{radius})^2$

*washer:* Area =  $\pi(\text{outer})^2 - \pi(\text{inner})^2$

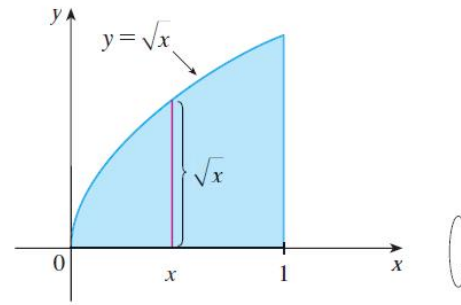
*square:* Area = (Height)(Length)

*triangle:* Area =  $\frac{1}{2}$  (Height)(Length)

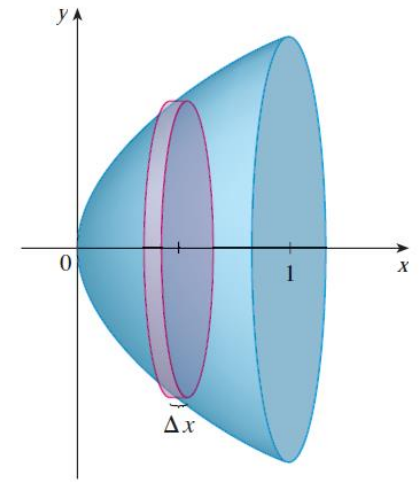
3. Integrate the area formula.

**Note:** If you draw a line at the perpendicular cut, the *axis you cut across is the variable you are using!*

*Example:* Consider the region,  $R$ , bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 1$ . Find the volume of the solid obtained by rotating  $R$  about the **x-axis**.



(a)

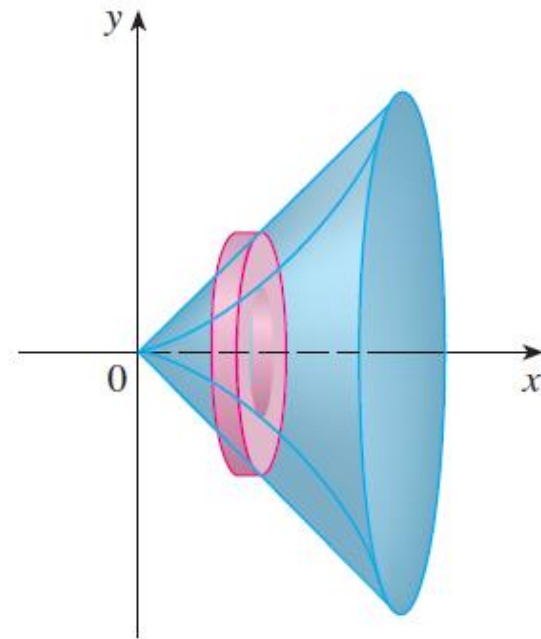


(b)

*Example:* Consider the region,  $R$ ,  
bounded by  $y = \sqrt{x}$ ,  $x = 0$ , and  $y = 1$ .  
Find the volume of the solid obtained  
by rotating  $R$  about the  **$y$ -axis**.

*Example:* Consider the region,  $R$ , bounded by  $y = x$  and  $y = x^4$ . Find the volume of the solid obtained by rotating  $R$  about the **x-axis**.

1. Draw and label!
2. Cross-sectional area?
3. Integrate area.



*Example:* Consider the region,  $R$ ,  
bounded by  $y = x$  and  $y = x^4$ .

( $R$  is the same as the last example).

(a) Now rotate about the horizontal  
line  $y = -5$ . What changes?

(b) Now rotate about the horizontal  
line  $y = 10$ . What changes?

*Example:*

Set up an integral for find the volume obtained by rotating the region bounded by  $y = x^3$ ,  $y = 8$ , and  $x = 0$  about the vertical line  $x = -10$ .

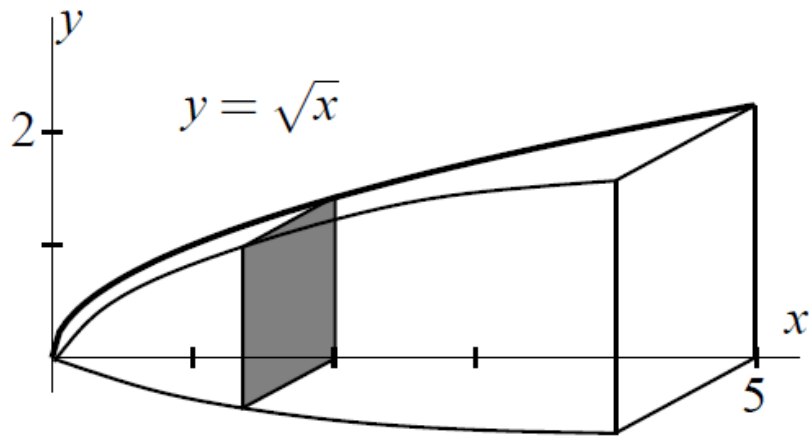


*Example:*

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

## Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

## This method has a major limitation:

6.2 method about  $x$ -axis, must use  $dx$ .

6.2 method about  $y$ -axis, must use  $dy$ .

What if the regions is rotated about the  $x$ -axis and we need to use  $dy$ ?  
(about  $y$ -axis and we need to use  $dx$ ?)

**In these cases, 6.2 “Cross-sectional slicing” wouldn’t work!**

We need another method.

That is what we will do in 6.3.